# Supporting Document 

Number Facts Grids

# Suggestions for Getting the Most Out of Blackline Masters 

## Background

## TWO VITAL TOPICS

Two mathematics topics that are essential for children to master if they are to have confidence and security on this subject. They are Number Facts and Place Value. In regards to Number Facts, children need to have instant recall. When I talk about Place Value I mean the list of skills and understandings that cover the basics of numeration, e.g. ordering numbers, reading numbers, counting, index notation, expanding numbers etc. While I am discussing Place Value, I would include the list of pre-number concepts as well. Success in these two topics in primary/elementary school will almost guarantee success in higher levels of mathematics.

## DRILLING NUMBER FACTS

In many occupations practising a skill will be essential for reaching the required level of proficiency. 'Drill and practice' in classroom mathematics in some countries used to be a 'dirty word'. In other words, skill and practice were discouraged, and children were expected to work out their number facts from first principles. For instance, if a child knows how to double numbers then to arrive at the product of 4 and 8 a child could double 4 ( 8 ), then double 8 (16), then double 16 (32). So $4 \times 8=32$ ).

## THE BENEFITS OF INSTANT RECALL OF NUMBER FACTS

- Students will have more mathematical confidence.
- They can concentrate on learning a new process, e.g. long division, or solve a mathematical problem, without sidetracking to work out their number facts.
- They will develop mental powers generally.
- They will improve their memory and recall.
- They will, progressively, develop more positive attitudes towards mathematics.


## Using Blackline Masters mblm001a, b, c, d, e \& f

This set of sheets was designed to help students achieve instant recall of number facts. Mblm001b is a student worksheet I used for drilling number facts for addition, subtraction, multiplication and division. It is suitable for use in the upper primary/elementary grades and higher if students are not yet competent. I see no reason why, by the end of grade 4, the average student couldn't master all their number facts. I used these Blackline Masters with years 5,6 and 7 as I was usually appointed to teach upper primary/elementary classes.

## MY METHODOLOGY

1. I informed the students which operation/process they would use.
2. Each student was given a copy of mblm001b face down.
3. For the addition, subtraction and multiplication squares students were to start from the top left and work to the right, working down the square in order filling in the squares. For division, they were to start from the top left and work down the columns completing each equation/number sentence in order.
4. I told the students they should keep work covered, and would have a set time to work. When I said, "Stop", students stopped writing and drew a dark, vertical line on the right-hand side of the last answer. Then they were to put pens down.
5. Using the clock with a second hand, which was on the front wall, I said, "Begin." when the second hand was on the twelve. Students turned their sheets over and began filling in the squares as quickly as they could.
6. Any student that completed the square before the time was up was to check the clock and write their work time in digital form, e.g. 2:35 (two minutes, thirty-five seconds) to the far right of the appropriate operation/process beneath 'Results'. They were then to put their hands on their heads and not talk. I informed the class when as each minute passed, e.g. "One minute...two minutes."
7. After three minutes I said, "Stop." Once all students who hadn't finished had marked their place They put their hands on their heads.
8. I then told the students to take their hands off their heads, and that those who had not finished had a reasonable time to complete their answers while the other students went on with other work.
9. Once all students had finished their answers, they would exchange papers for marking. As I read answers from the answer sheet, mblm001c, students checked each other's responses. Students circled any wrong answers and said, "No." Students accounted wrong any numbers that were illegible or ambiguous.
10. I asked students are to say, "No" out loud, to give me a quick evaluation of the number facts that needed general revision. Circling the wrong answers pointed out to students which number facts they needed to learn.

## THE RESULTS

At the end of one term, almost all of my students were completing the multiplication square in less than three minutes with the quickest completing it in about 1:30 (That's one minute and thirty seconds, not one hour and thirty minutes). :-)

In this way, I evaluated students in their mastery of number facts. Students also had regular homework and in-class work to help them learn their tables.

Blackline Masters mblm001a \& f can be used for extra practice or for making up squares with different top and side numbers.

## INSTANT RECALL

By the time students are in grade four most would have learned by rote the months of the year and the days of the week. Just test it for yourself. Say to your class that you are going to say three words and you want them to call out the next word in the sequence. Take note of how many make the
correct response. Here are some examples to try:

- "Sunday, Monday, Tuesday..."
- "Wednesday, Thursday, Friday..."
- "January, February, March..."
- "July, August, September..."

If young children can learn the days of the week and the months of the year, they can master their multiplication tables as well. Frequent repetition is the key. One way is to repeat each table, e.g. "One two is two. Two tows are four...ten twos are twenty." Once learned well, they will know it for life, or up to the time, their memory starts to go to pieces.

## MAKING MATHEMATICS EASIER FOR YOURSELF \& YOUR STUDENTS

Scroll down to learn two strategies that will encourage your students and lower your stress levels when students are learning multiplication facts.

## STRATEGY 1 - GETTING THE MOST OUT OF ARRAYS

Students need to think in number families when learning number facts. For instance, with $2+3=5$ we create a family of numbers, 2,3 and 5 . Learning that these numbers are 'related' by addition and subtraction then the number family can be can be used to determine $3+2=5,5-3=2$ and $5-2=3$. In other words, given these understandings, a student does not have to learn their subtraction facts by rote. It reduces the number of facts the student needs to learn by $50 \%$.

It is the same with multiplication and division. Understanding a few concepts can make it easier for students learning number facts. Consider the array below:
$6 \times 3$ Array Using Bread Bag Seals


The $6 \times 4$ array establishes the number family 6,3 and 18 . For each array, students should be able to 'see' the following facts: $6 \times 3=18,3 \times 6=18,18 \div 6=3,18 \div 3=6,1 / 3 \times 18=6$ and $1 / 6 \times 18=3$. Once a student knows $1 / 3 \times 18=6$, "One-third of eighteen", then they can work out that $2 / 3 \times 24=12$. Other fraction facts using the same number family can also be determined, e.g. $3 / 6 \times 18=9$ and $5 / 6 \times 18=15$.

Students can quickly learn to compile the six number facts, two each for multiplication, division and fractions, for all arrays up to $10 \times 10$ or in some countries $12 \times 12$. Students will need time to practice this procedure and see the benefits of putting in a little effort. With this understanding, students will not have to 'learn' their division or related fraction facts up to 100.

If students understand the commutative law of multiplication and addition, or 'turn a-rounds', they will reduce the number of facts they need to learn by $50 \%$. The use of the array is an effective way for students to know that ' 3 lots of 6 ' is the same as ' 6 lots of 3 ' as in $3 \times 6=18$ and $6 \times 3=18$.

Now let's move on to Strategy 2.

## STRATEGY 2 - BRINGING IT ALL TOGETHER TO ENCOURAGE YOUR STUDENTS

Students need a copy of the 'Multiplication Facts' figure below. Blackline master mblm001d has two of these figures. Students will be using this figure to indicate which facts they will not have to learn. The students are not to place crosses in the top row or the left column. They are only to cross the boxes that contain products. We begin with 121 facts, the number in parentheses at the end of each step is the number of facts left uncrossed after 'crossing' the number of facts for steps up to that point.

| X ultiplication Facts |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  |  |
| 2 | 0 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |
| 3 | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |  |  |  |
| 4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |  |  |  |
| 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |  |  |  |
| 6 | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |  |  |  |
| 7 | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |  |  |  |
| 8 | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |  |  |  |
| 9 | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |  |  |  |
| 10 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |  |  |  |

Extracted from Blackline Master mblm001d

The following steps involve the facts, or products, that a student in upper primary (elementary) grades are likely to know already. The object of the exercise is for students to discover they don't have thousands of facts to learn, but that the more concepts they understand, the less they have to learn.

Step 1. Most students already understand the commutative law (turn a-rounds). If they do, they can place a cross, corner to corner, in the squares above the diagonal from 0 to 100 (top left to bottom right), i.e. above the squares of the numbers $0,1,4,9,16 \ldots 100$. The crossed facts are facts they will not need to learn if they understand the principle that $3 x 7$ is equal to $7 \times 3$. (121-55=66 remaining)

Step 2. Most students understand that the result is zero if anything is multiplied by zero. If they do, they can place a cross in the remainder of product squares that are multiples of zero. (66-11=55 remaining)

Step 3. Most students know their one times table (multiples of 1 up to 10). If they do, they are to place a cross in the remaining multiples of 1. (55-10=45 remaining)

Step 4. Do the same for the remaining multiples of 2 up to 20. (45-9=36 remaining)

Step 5. Do the same for the remaining multiples of 10 up to 100. (36-10=26 remaining)

Step 6. Do the same for the remaining multiples of 5 up to 50 . (26-8=18 remaining)

Step 7. It is easy to learn the nine times table. The digits of the products of nine add up to nine, the first digit or the product is one less than the number that is multiplied by nine, e.g. $3 \times 9=27$ notice the first digit of the product is 2 , one less than the 3 . So that the sum of the digits is 9 , we place a 7 next to the 2 . (18-6=12 remaining) Only 12 harder multiplication facts to learn! :-)

Teachers need to point out to their students the following:

- If they already know the simpler facts above, then they only have 12 multiplication facts to learn.
- If they understand how to derive division facts from arrays, i.e. obtaining the six basic number facts from the number families for multiplication, they will not have to learn any division facts. A bonus is knowing fraction facts that relate to their multiplication facts.
- If they understand the commutative law of addition (turn a-rounds), they will not have to learn any subtraction facts.

Blackline Master mblm001e could be used for students to write in the facts they know well. Multiplication and addition facts flashcards, with the expression, e.g. $4 \times 5$, on the front and the product 20 on the back, are useful resources for learning and checking number facts. Saying their tables by rote, e.g. "One five is five, two fives are ten, three fives are fifteen...ten fives are fifty." I found very effective. I used to have competitions to see which student could say them in the least amount of time.

By<br>Richard Warden

